Lec 20: 04/02/2012

Accretion Columns:

If the compact object at the center is strongly magnetized, the disk will not extend all the way down to the stellar surface. The radial infall will be disrupted some distance away from the star. This is intuitively expected as magnetic fields tend to whirl particles around.

The magnetic field may be quite complex near the surface of the compact object. However, all of the multipole moments higher than dipole fall off very quickly away from the star. It therefore suffices to keep the magnetic dipole moment $\vec{m}$, which results in:

$$\vec{B} = \frac{3\hat{n}(\hat{n}\cdot\vec{m}) - \vec{m}}{r^3}$$

On the horizontal plane, we have:

$$B(R) = B_* \left(\frac{R_*}{R}\right)^3$$

Here B_* is the magnetic field at the polar cap $(R = R_*)$.

A simple criterion for dominance of the magnetic field over thermal motion is that the energy density in the magnetic field becomes larger than the thermal energy density:

$$\frac{1}{8\pi} B^2 \gtrsim \frac{1}{2} \rho v_\varphi^2 = \frac{1}{2} \frac{GM\rho}{R} \qquad (v_\varphi = R\Omega_k(R))$$

From previous lectures, we have:

$$\rho \sim \frac{\dot{M}}{12\pi} \frac{1}{3\alpha} \left(\frac{\mu}{R_g T}\right)\left(\frac{GM}{R^3}\right)^{\frac{1}{2}}$$

The magnetic radius R_* is then found to be:

$$R_m \simeq B_* R_*^3 \left(\frac{R_g T}{\mu}\right)^{3/4} \frac{1}{GM} \left(\frac{3}{4}\alpha\right)^{\frac{1}{2}} \dot{M}^{-\frac{1}{2}}$$

It can be written as:

$$R_m \simeq (30\ km)\, \alpha^{\frac{1}{2}} \left(\frac{B_*}{10^{12}\, G}\right)\left(\frac{R_*}{10^6\, cm}\right)^3 \left(\frac{T}{10^6\, K}\right)^{3/4} \left(\frac{M_\odot}{M}\right)\left(\frac{\dot{M}}{10^{16} g\, s^{-1}}\right)^{-\frac{1}{2}}$$

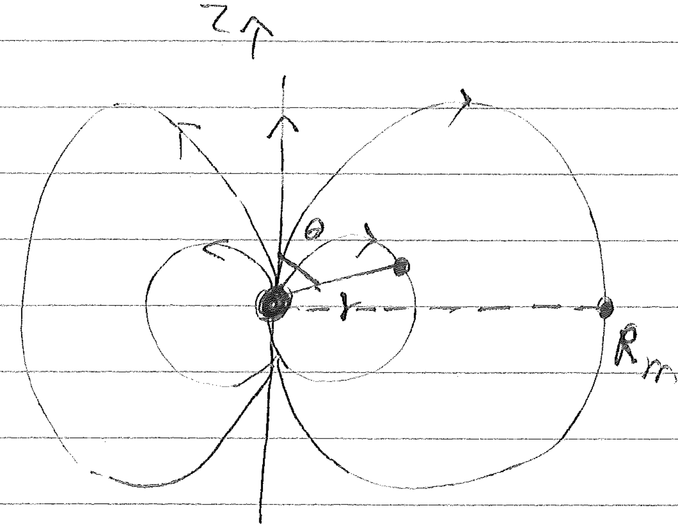
For a neutron star, when the parameters take their natural values used for normalization, we have $R_m \sim 3R_*$.

This suggests that the radial flow of material and the disk are disrupted relatively far away from the surface of the star.

Accretion Columns in Magnetic Cataclysmic Variables:

Once the gas reaches $R \sim R_m$, it is funneled toward the stellar polar caps along the magnetic field line. This is the only possibility for motion when the magnetic field is very strong. This cropping of the disk is seen in both systems where the primary is a magnetized white dwarf or a pulsar. Magnetic Cataclysmic Variables (mCVs), in which the primary is a magnetized white dwarf, are better understood than X-ray pulsars (their neutron star counterparts). Not only the physical modeling is more straightforward in this case, but also there is more observational evidence available for comparison with theory.

A dipole field line is described by the relation:



$r = \text{Const.} \times \sin^2\theta$

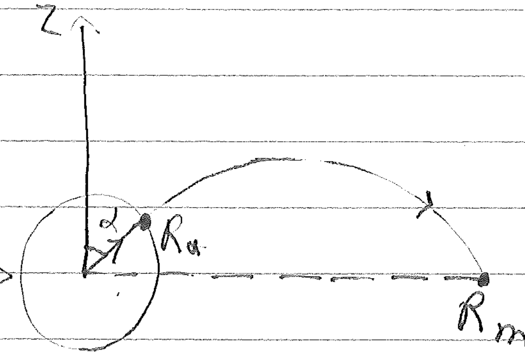
$r = R_m$ at $\theta = \frac{\pi}{2}$

Thus:

Const. $= R_m$

The outer angle of impact on the white dwarf's surface is:

$$\delta = \sin^{-1}\left(\frac{R_*}{R_m}\right)^{\frac{1}{2}}$$



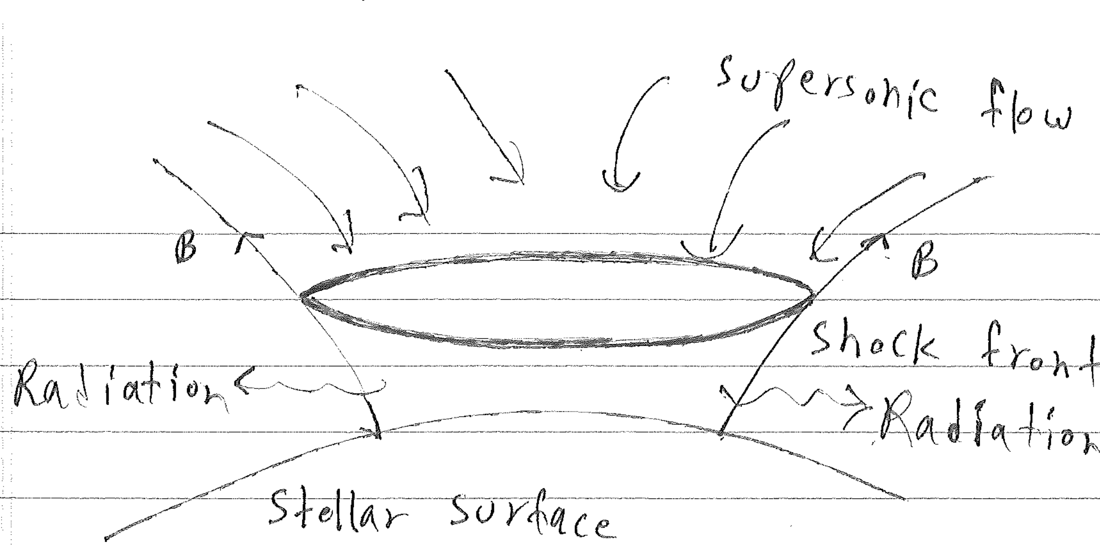
$R_* \sim 10^9$ cm, $B_* \sim 2 \times 10^7$ G, $T \sim 10^5$ K $\Rightarrow$

$R_m \sim 10^{10}$ cm

Thus in the case of a white dwarf:

$\delta \sim 20^\circ$

Note that magnetic field lines that thread the disk beyond R_m will feed the termination full at smaller δ, and hence make the region $\theta \leq \delta$ full.



For $\delta \sim 20°$, the polar cap area beneath the flow is $\sim \frac{1}{40}$ of the total stellar surface area. Since both polar caps can be active, a fraction $f \sim \frac{1}{20}$ of the entire white dwarf's surface participates in the accretion.

Inside the funnel, the falling matter reaches a velocity:

$$v_{ff} = \left(\frac{2GM}{R_*}\right)^{1/2} \sim 5 \times 10^8 \text{ cm s}^{-1}$$

The thermal velocity of electrons at temperatures relevant for a white dwarf is $v_{th} = \left(\frac{kT}{m_e}\right)^{1/2} \sim 10^8$ cm s^{-1} (and much smaller than this for protons). This is roughly the speed of sound in the medium, which implies that the inflow is supersonic. As a result, it produces a shock that heats of the material in the region behind the shock.

Recall that across a strong shock the relative velocity drops by a factor of 4. Conservation of mass then implies an increase in the density by the same factor. The density in the shocked region will therefore be:

$$\rho_{shock} \simeq 4 \frac{\dot{M}}{4\pi f R_*^2} \left(\frac{2GM}{R_*}\right)^{-\frac{1}{2}} \sim 10^{-10} \, g \, cm^{-3}$$

The corresponding temperature is;

$$T_{shock} \sim \frac{m_H}{3K} \left(\frac{3 v_{ff}}{4}\right)^2 \sim 6 \times 10^8 \, K$$

The plasma is thus heated up by the shock considerably. It then cools off quickly as photons leave the system without being trapped. This can be seen by finding the optical depth in the transverse direction;

$$\tau_\perp \sim \left(\frac{\rho_{shock}}{\mu m_H}\right) \sigma_T \, f^{\frac{1}{2}} R_* \sim 0.03$$

The medium is therefore optically thin.

The overall spectrum from accretion column of a white dwarf has various components. The dominant one is a hard, optically thin bremsstrahlung radiation in the X-ray. There is also cyclotron emission in the UV (since there is a magnetic field), as well as a soft blackbody X-ray radiation that comes from the reprocessing of hard X-rays that penetrate below the stellar surface.

Accretion Columns in X-ray Pulsars:

The main complication arising in the case of X-ray pulsars is the larger radiation pressure in this case. Since the accreted plasma is funneled onto smaller polar cap regions, "f" will be much smaller in this case (at least by an order of magnitude). The radiative flux will therefore be larger than an isotropically emitting source by a factor of $\frac{1}{f}$. This implies that even with an accretion rate $L_{acc} \sim 0.1\, L_{edd}$, the effective luminosity out of the funnel can be comparable to L_{edd}. Once this happens, radiation pressure significantly affects accretion.

A second complication is that the transverse optical depth $\sigma_\perp$ is much larger in the case of X-ray pulsars. Since $\sigma_\perp \propto n_e R_*$, while $n_e \propto R_*^{-2}$, then $\sigma_\perp \propto R_*^{-1}$. Therefore $\sigma_\perp$ in the case of a neutron star is larger than that for a white dwarf by a factor of $\sim 10^3$, which results in $\sigma_\perp \sim 30$, hence an optically thick medium.

With an Eddington flux being produced within the funnel, and the medium being optically thick, one may expect to have instabilities. Numerical simulations confirm the view that the accretion column in these circumstance must be unstable.